CAPACITY EXPANSION INVESTMENT PLANNING MODEL

CHE 5480- University of Oklahoma
School of Chemical Engineering and Materials Science

Prof. M. Bagajewicz

Planning under Uncertainty (Stochastic Model)

Characteristics of Two-Stage Stochastic Optimization Models

Philosophy

- Maximize the *Expected Value* of the objective over all possible realizations of uncertain parameters.
- Typically, the objective is *Profit* or *Net Present Value*.
- Sometimes the minimization of *Cost* is considered as objective.

Uncertainty

- Typically, the uncertain parameters are: *market demands, availabilities, prices, process yields, rate of interest, inflation, etc*.
- In Two-Stage Programming, uncertainty is modeled through a finite number of independent *Scenarios*.
- Scenarios are typically formed by *random samples* taken from the probability distributions of the uncertain parameters.

Characteristics of Two-Stage Stochastic Optimization Models

- **First-Stage Decisions**
  - Taken before the uncertainty is revealed. They usually correspond to structural decisions (not operational).
  - Also called "Here and Now" decisions.
  - Represented by "Design" Variables.
  - Examples:
    - To build a plant or not. How much capacity should be added, etc.
    - To place an order now.
    - To sign contracts or buy options.
    - To pick a reactor volume, to pick a certain number of trays and size the condenser and the reboiler of a column, etc

Characteristics of Two-Stage Stochastic Optimization Models

Second-Stage Decisions

- Taken in order to adapt the plan or design to the uncertain parameters realization.
- Also called "Recourse" decisions.
- Represented by "Control" Variables.
- Example: the operating level; the production slate of a plant.

- Sometimes first stage decisions can be treated as second stage decisions. In such case the problem is called a multiple stage problem.

Two-Stage Stochastic Formulation

Let us leave it linear because as is it is complex enough.!!!

LINEAR MODEL SP

$$Max \sum_s p_s q_s^T y_s - c^T x$$

Recourse Function **First-Stage Cost**

$s.t.$

$$Ax = b$$ **First-Stage Constraints**

$$T_s x + W y_s = h_s$$ **Second-Stage Constraints**

$$x \geq 0 \quad x \in X$$

$$y_s \geq 0$$

Technology matrix

First stage variables

Second Stage Variables

Recourse matrix (Fixed Recourse)

Sometimes not fixed (Interest rates in Portfolio Optimization)

***Complete recourse*:** the recourse cost (or profit) for every possible uncertainty realization remains finite, independently of the first-stage decisions (x).

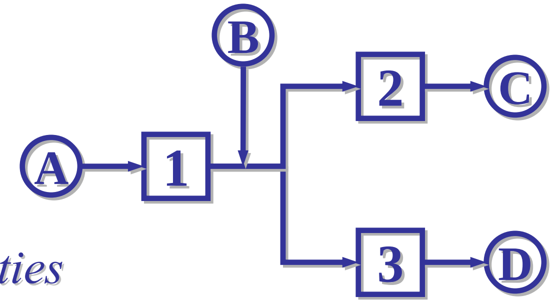
***Relatively complete recourse*:** the recourse cost (or profit) is feasible for the set of feasible first-stage decisions. This condition means that for every feasible first-stage decision, there is a way of adapting the plan to the realization of uncertain parameters.

We also have found that one can sacrifice efficiency for certain scenarios to improve risk management. We do not know how to call this yet.

Process Planning Under Uncertainty

GIVEN:

- **Process Network**
  - *Set of Processes*
  - *Set of Chemicals*



- **Forecasted Data**
  - *Demands & Availabilities*
  - *Costs & Prices*
  - *Capital Budget*

DETERMINE:

- **Network Expansions**
  - *Timing*
  - *Sizing*
  - *Location*
- **Production Levels**

OBJECTIVES:

- **Maximize Expected Net Present Value**
- **Minimize Financial Risk**

Process Planning Under Uncertainty

Design Variables: to be decided before the uncertainty reveals

$$x = \{ Y_{it}, E_{it}, Q_{it} \}$$

Y: Decision of building process *i* in period *t*
E: Capacity expansion of process *i* in period *t*
Q: Total capacity of process *i* in period *t*

Control Variables: selected after the uncertain parameters become known

$$y_s = \{ S_{jlts}, P_{jlts}, W_{its} \}$$

S: Sales of product *j* in market *l* at time *t* and scenario *s*
P: Purchase of raw mat. *j* in market *l* at time *t* and scenario *s*
W: Operating level of of process *i* in period *t* and scenario *s*

MODEL

LIMITS ON EXPANSION $Y_{it}E_{it}^{L} \leq E_{it} \leq Y_{it}E_{it}^{L}$ $i=1,\ldots,NP \quad t=1,\ldots,NT$

TOTAL CAPACITY IN EACH PERIOD $Q_{it} = Q_{i(t-1)} + E_{it}$ $i=1,\ldots,NP \quad t=1,\ldots,NT$

LIMIT ON THE NUMBER OF EXPANSIONS $\sum_{t=1}^{NT} Y_{it} \leq NEXP_{i}$ $i=1,\ldots,NP$

LIMIT ON THE CAPITAL INVESTMENT $\sum_{i=1}^{NP} \left(\alpha_{it}E_{it} + \beta_{it}Y_{it}\right) \leq CI_{t}$ $t=1,\ldots,NT$

Y_{it}: An expansion of process I in period t takes place (Yit=1), does not take place (Yit=0)
E_{it}: Expansion of capacity of process i in period t.
Q_{it}: Capacity of process i in period t.

I : Processes i,=1,…,NP
J : Raw mat./Products, j=1,…,NC
T: Time periods. T=1,…,NT
L: Markets, l=1,..NM

$NEXP_t$: maximum number of expansions in period t
α_{it} : Variable cost of expansion for process i in period t
β_{it} : Fixed cost of expansion for process i in period t
E_{it}^{L}, E_{it}^{U} : Lower and upper bounds on a process expansion in period t

MODEL

UTILIZED CAPACITY IS LOWER THAN TOTAL CAPACITY

$$W_{its} \le Q_{it} \qquad i=1,\ldots,NP \quad t=1,\ldots,NT$$

MATERIAL BALANCE

$$\sum_{l=1}^{NM} P_{jlts} + \sum_{i=1}^{NP} \eta_{ij} W_{its} \le \sum_{l=1}^{NM} S_{jlts} + \sum_{i=1}^{NP} \mu_{ij} W_{its} \qquad i=1,\ldots,NP \quad t=1,\ldots,NT, \forall s$$

BOUNDS

$$a^L_{jlts} \le P_{jlts} \le a^U_{jlts} \qquad d^L_{jlts} \le S_{jlts} \le d^U_{jlts} \qquad j=1,\ldots,NC,\ t=1,\ldots,NT,\ l=1,\ldots,NM, \forall s$$

NONNEGATIVITY

$$E_{it}, Q_{it}, W_{its}, P_{jlts}, S_{jlts} \ge 0 \qquad \forall i,j,l,t$$

INTEGER VARIABLES

$$Y_{it} \in \{0,1\}$$

Y_{it}: An expansion of process I in period t takes place (Yit=1), does not take place (Yit=0)
E_{it}: Expansion of capacity of process i in period t.
Q_{it}: Capacity of process i in period t.
W_{it}: Utilized capacity of process i in period t.
P_{jlt} : Amount of raw material/intermediate product j consumed from market l in period t
S_{jlt} : Amount of intermediate product/product j sold in market l in period t

I : Processes i,=1,…,NP
J : Raw mat./Products, j=1,…,NC
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a^L_{jlts}, a^U_{jlts} :*Lower and upper bounds on availability of raw material j in market l in period t, scenario s*

d^L_{jlts}, d^U_{jlts} : *Lower and upper bounds on demand of product j in market l in period t, scenario s*

MODEL

OBJECTIVE FUNCTION

$$Max\ \ NPV = \underbrace{\sum_{s} p_s \left\{ \sum_{t=1}^{NT} L_t \left(\sum_{l=1}^{NM} \sum_{j=1}^{NC} \left(\gamma_{jlts} S_{jlts} - \Gamma_{jlts} P_{jlts} \right) - \sum_{i=1}^{NP} \delta_{its} W_{its} \right) \right\}}_{DISCOUNTED\ REVENUES} - \underbrace{\sum_{i=1}^{NP} \sum_{t=1}^{NT} \left(\alpha_{it} E_{it} + \beta_{it} Y_{it} \right)}_{INVESTMENT}$$

DISCOUNTED REVENUES *INVESTMENT*

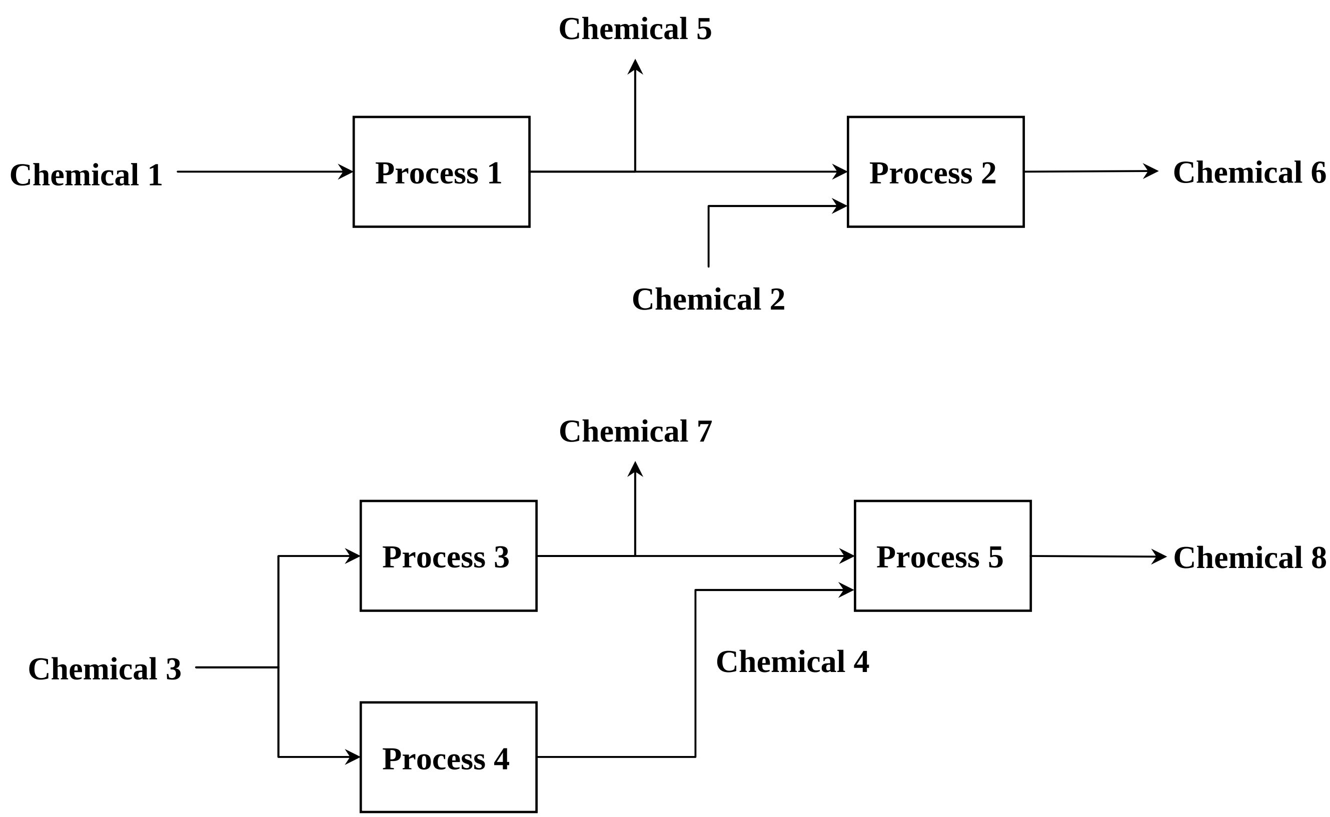
Y_{it}: An expansion of process I in period t takes place (Yit=1), does not take place (Yit=0)
E_{it}: Expansion of capacity of process i in period t.
W_{it}: Utilized capacity of process i in period t.
P_{jlt} : Amount of raw material/interm. product j consumed from market l in period t
S_{jlt} : Amount of intermediate product/product j sold in market l in period t

I : Processes i,=1,…,NP
J : Raw mat./Products, j=1,…,NC
T: Time periods. T=1,…,NT
L: Markets, l=1,..NM

γ_{jlt} : Sale price of product/intermediate product j in market l in period t
Γ_{jlt} : Cost of product/intermediate product j in market l in period t
δ_{it} : Operating cost of process i in period t
α_{it} : Variable cost of expansion for process i in period t
β_{it} : Fixed cost of expansion for process i in period t
L_t : Discount factor for period t

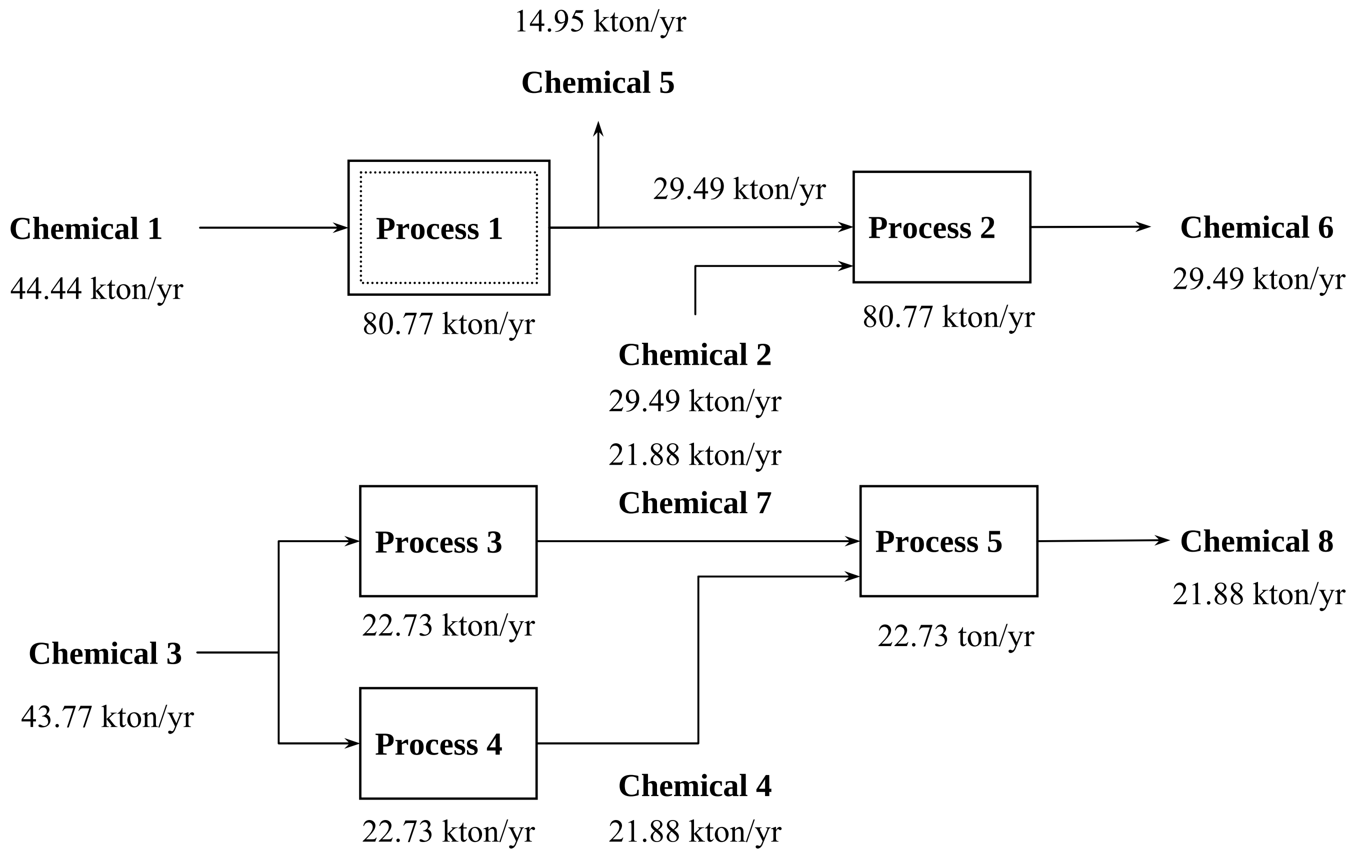
Example

- Uncertain Parameters: Demands, Availabilities, Sales Price, Purchase Price
- Total of 400 Scenarios
- Project Staged in 3 Time Periods of 2, 2.5, 3.5 years



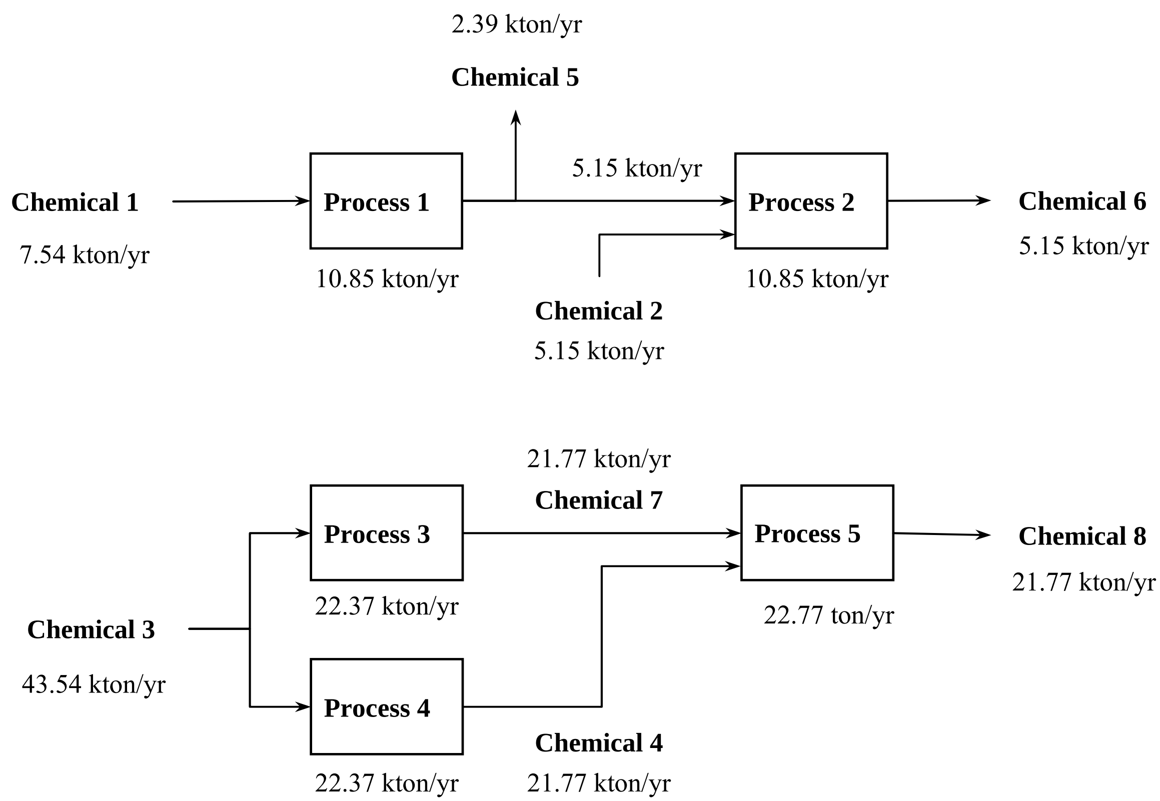
Example – Solution with Max ENPV

Period 3
3.5 years



Example – Solution with Min DRisk(Ω=900)

Period 3
3.5 years



Example – Solution with Max ENPV